CUBE-LIKE COMPLEXES AND POINCARÉ-MIRANDA THEOREM

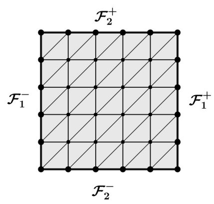
1. ABSTRACT

In 1883 Henri Poincaré announced the following theorem without the proof: *If f is continuous function $f = (f_1, f_2, \dots, f_n) : I^n \to \mathbb{R}^n$, such that $f_i(I_i^-) \subset (-\infty, 0], f_i(I_i^+) \subset [0, \infty)$, $I_i^- := \{x \in I^n : x(i) = -1\}$, $I_i^+ := \{x \in I^n : x(i) = 1\}$, then there is $c \in I^n$ such that $f(c) = 0$.*

In 1940 Miranda rediscovered the Poincaré theorem and showed that it is equivalent to the Brouwer fixed point theorem.

The aim of this presentation is to show a generalization of the Poincaré-Miranda theorem as well as its parametric extension. We introduce the new class of complexes that we call n-cube-like complexes. This class is generalization of n-dimensional cubes and it consists of n-dimensional cubes, but also different complexes, for example: the Möbius strip, solid torus or cube with holes. We show that the Poincaré theorem holds for n-cube-like polyhedrons but our examples demonstrate that there exist n-cube-like polyhedrons without fixed point property.

Example 1. The cube I^n with an arbitrary triangulation is an n-cube-like polyhedron.



Example 2. Example of a 2-cube-like polyhedron without a fixed point property.